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2008 J. Phys. A: Math. Theor. 41 065301

(http://iopscience.iop.org/1751-8121/41/6/065301)

IOP PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND THEORETICAL

J. Phys. A: Math. Theor. **41** (2008) 065301 (8pp) doi:10.1088/1751-8113/41/6/065301

Influence of the topology in EPR correlations

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Received 31 May 2007, in final form 19 December 2007

Published 29 January 2008

Online at stacks.iop.org/JPhysA/41/065301

Abstract

In this work, we verify the influence of the topology in Einstein–Podolsky–Rosen (EPR) correlations in the spacetime of a cosmic string. We show by means of a Wigner rotation that the presence of this topological defect breaks the spin anti-correlations of the EPR pairs. We also show that a perfect correlation can be obtained if we take into account the relativistic effect arising from the acceleration and the spacetime topology.

PACS numbers: 03.65.Ud, 03.67.−a, 04.20.−q, 02.40.−k

1. Introduction

Over the past 70 years of the remarkable Einstein, Podolsky and Rosen (EPR) paper [1], the study of non-locality and entanglement in quantum mechanics ranges from purely philosophical problems to quantum cryptography [2, 3], computation and teleportation [4]. In 1964, Bell [5], by means of his theorem on reality and locality, showed that nature indeed seems to be non-local as far as non-relativistic quantum mechanics is concerned. In this context, the EPR-correlated states are now widely accepted as a vital resource on quantum information, such as quantum cryptography and quantum computation.

The question of non-locality still remains to be answered in other arenas, like in relativistic dynamics. In this way, the analysis of the EPR correlations in several relativistic physical situations is quite interesting in order to obtain the complete description of quantum communication in the relativistic regime.

In recent years, a numbers of papers [6–16] analyzed the relativistic effects in quantum information, such as a discussion of how the entanglement is affected by the Lorentz transformations in the regime of special relativity. Due to the importance of the subject, these works inspire the study of EPR anti-correlations in the point of view of general relativity. In this context, the pioneer work is due to von Borzeszkwski and Mensky [17] who have investigated the EPR paradox using the parallel transport matrix. Recently, several articles have investigated the influence of curved backgrounds in quantum information and computation

[18–22]. Terashima and Ueda [20, 23] studied the influence of a gravitational field on the properties of the EPR-correlated states by means of general relativity. In particular, they studied the EPR correlations for a spin in an orbit around a Schwarzschild black hole. They concluded that acceleration and gravity deteriorate the perfect anti-correlation of the EPR pairs of spin in the same direction and, apparently, decrease the degree of violation of Bell's inequality. They also claimed that the perfect anti-correlation of the spin singlet state in the EPR correlation is maintained by an appropriate choice of the spin-measurement directions depending on the velocity of the particle, the curvature of the spacetime and the position of the observers. In recent articles [24, 25], a series of experiments were proposed using space infrastructure with specific emphasis on the satellite-based distribution of entangled photon pairs. The authors of [24, 25] argued that these experiments can be used concerning special and general relativistic effects on the quantum entanglement.

In this paper, we are interested in the influence of the topology in EPR correlations. We will analyze the spacetime of a topological defect, particularly a cosmic string. The geometry of a cosmic string is characterized by a conical singularity in the origin of the coordinate system. Thus, we can affirm that the spacetime of a cosmic string is locally, but not globally, flat. This appears due to the presence of a singularity in the origin. Because of the simplicity of this model, we use it to analyze the influence of topology in a EPR *gedankenexperiment*.

This paper is organized in the following form. In section 2, we will make a brief exposition on cosmic string properties. In section 3 we present a mathematical formulation for a spin-1/2 particle, in the context of EPR correlations, in the presence of a gravitational field, and we study the precession of a spin using the rotation of Wigner. Finally, in section 4 we present the concluding remarks.

2. Cosmic string spacetime

Cosmic strings are linear defects produced in the early universe due to the symmetry breaking phase transitions, involved in the process of cooling of the universe. It is well known that the spacetime produced by a thin, infinite, straight cosmic string has no Newtonian potential [26] and cannot induce curvature (locally the curvature vanishes everywhere except at the source). However, there are some global non-trivial topological effects associated with this spacetime which can be measured [27]. This geometry also presents quantum effects, which have been studied, either in the context of simple cones or in the context of cosmic string [27].

The simplest form of representing a cosmic string is by an infinite straight line. Because of its simplicity, this model presents a high degree of symmetry. The line element of a cosmic string is given by

$$ds^2 = -c^2\,dt^2 + d\rho^2 + dz^2 + \alpha^2\rho^2\,d\varphi^2, \tag{1}$$

where α is called the deficit angle and is defined as $\alpha = 1 - 4\mu G/c^2$ where μ is the linear mass density of the cosmic string. The azimuthal angle varies in the interval: $0 \leqslant \varphi < 2\pi$. The deficit angle can assume only values in which $\alpha < 1$ (for a disclination [28, 29], it can assume values greater than 1, which correspond to an anti-conical spacetime with negative curvature). This geometry possesses a conical singularity represented by the following curvature tensor:

$$R^{\rho,\varphi}_{\rho,\varphi} = \frac{1-\alpha}{4G\alpha}\delta_2(\vec{r}), \tag{2}$$

where $\delta_2(\vec{r})$ is the two-dimensional delta function. This behavior of the curvature tensor is denominated conical singularity [30]. The conical singularity gives rise to the curvature concentrated on the cosmic string axis. In all other places, the curvature is null.

It is well known from quantum field theory in curved spaces that the particle state is not defined uniquely, since the time coordinate to define a positive energy is not unique. This question emerges from the concept of particle creation in processes such as Hawking radiation. In general, in a curved spacetime a global time-like Killing vector field does not exist; therefore the direct association of a particle with a state of the quantum field is meaningless. However, if a local Killing vector can be defined in some region of this space, this allows us to find an interpretation for particles in this space. This question does not bring any problems for the case studied here because the geometry of the cosmic string is locally flat [31, 32] but not globally.

3. EPR correlations in a cosmic string background

Now, we return our attention to the main objective of this paper which is the investigation of EPR correlations in the cosmic string background. The aim of this paper is to investigate the influence of the topology in the EPR correlations. The topological defect contributes to change the topology of spacetime. The cosmic string spacetime has some characteristics of Minkowski spacetime since this spacetime is locally flat, but the conical singularity in $\rho = 0$ changes the topology of Minkowski spacetime. The study of relativistic and non-relativistic quantum dynamics in the presence of topological defects produces a series of interesting quantum effects; as an example, we can cite the gravitational Aharonov–Bohm effect.

We consider that there are two observers and an EPR source on the z plane, where their positions are given by the azimuthal angles $\varphi = \pm\Phi$ and $\varphi = 0$, respectively. We assume that they are static in the coordinate system (t, ρ, z, φ) at rest. First, the EPR source emits a pair of entangled particles in opposite directions. The 4-momentum of each particle is given, respectively, by

$$p^a_{\pm}(x) = (mc\cosh\xi, 0, 0, \pm mc\sinh\xi), \tag{3}$$

where the state that describes this system can be written as

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left\{\left|p^a_+(x), \uparrow; x\right\rangle\left|p^a_-(x), \downarrow; x\right\rangle - \left|p^a_+(x), \downarrow; x\right\rangle\left|p^a_-(x), \uparrow; x\right\rangle\right\}. \tag{4}$$

Our aim is to localize two observers in two points of spacetime to measure the behavior of the state (4) when the cosmic string is present. To make this, we consider that the particles move to a new point $\bar{x}^\mu = x^\mu + U^\mu(x)\,d\tau$ of spacetime, with $U^\mu(x)$ being the 4-velocity and τ the proper time of the particles. Thus, at this new point, the 4-momentum of the particles varies at each local referential frame and its expression is given by

$$p^a(\bar{x}) = p^a(x) + \delta p^a(x). \tag{5}$$

The variation $\delta p^a(x)$ originates in the dependence of the 4-velocity in relation to the point where the particle is located. The local reference frame of each particle is defined by the components of the non-coordinate basis $e^a = e^a_{\ \mu}(x)\,dx^\mu$. The components $e^a_{\ \mu}(x)$ are called tetrads and are defined by

$$g_{\mu\nu}(x) = e^a_{\ \mu}(x)e^b_{\ \nu}(x)\eta_{ab}, \tag{6}$$

with the Greek and Latin indices running over $\mu, \nu, \ldots = t, \rho, z, \varphi$ and $a, b, \ldots = 0, 1, 2, 3$, and the tensor $\eta_{ab} = \mathrm{diag}(-1, 1, 1, 1)$ is the Minkowski metric. The tetrads satisfy the relations

$$e^a_{\ \mu}(x)e^\mu_{\ b}(x) = \delta^a_{\ b}, \qquad e^\mu_{\ a}(x)e^a_{\ \nu}(x) = \delta^\mu_{\ \nu}. \tag{7}$$

When we perform a local Lorentz transformation in the tetrad field, we have the following transformation rule:

$$e^a_{\ \mu}(x) \longrightarrow \bar{e}^a_{\ \mu}(x) = \Lambda^a_{\ b}(x)e^a_{\ \mu}(x), \tag{8}$$

where $\Lambda^a_b(x)$ is a Lorentz matrix. Hence, the variation in the 4-momentum is made by the application of successive infinitesimal Lorentz transformations at each local referential frame of the particles and provides the formation of the Wigner rotation. The Wigner rotation allows us to observe the behavior of spins in the state given in (4) because the Wigner rotation is made in the reference frame in which particles are at rest. The result of Wigner rotation on the spins is a precession on its direction of measurement in the local reference frame of the particle at rest in relation to its initial configuration. The state of the particles (4) under a local Lorentz transformation $\Lambda^a_b(x)$ is

$$U(\Lambda(x))|p^a(x),\sigma;x\rangle = \sum_{\sigma'} D^{1/2}_{\sigma'\sigma}(W(x))|\Lambda p^a(x),\sigma';x\rangle, \tag{9}$$

with σ being the spins of the particles and the Wigner rotation given by $W^a_b(x) \equiv W^a_b(\Lambda(x), p(x))$. In this way, the variation $\delta p^a(x)$ can be given as

$$\delta p^a(x) = \delta p^\mu(x) e^a_\mu(x) + p^\mu(x)\delta e^a_\mu(x). \tag{10}$$

The effective change in the momentum is $\delta p^\mu(x) = m a^\mu(x)\,\mathrm{d}\tau$, with $a^\mu(x)$ being the 4-acceleration of the particles and the variation in the local reference frame is $\delta e^a_\mu(x) = -U^\nu(x)\omega^a_{\nu b}(x)e^b_\mu(x)\,\mathrm{d}\tau$.

Now, we need to define the local reference frame and the motion of the particle in the cosmic string background if we want to observe the evolution of the state (4) in this background. Hence, with the line element given in expression (1), we choose our local reference frame through the 1-form dual basis as

$$e^0 = c\,\mathrm{d}t, \tag{11a}$$

$$e^1 = \mathrm{d}\rho, \tag{11b}$$

$$e^2 = \mathrm{d}z, \tag{11c}$$

$$e^3 = \alpha\rho\,\mathrm{d}\varphi, \tag{11d}$$

where $e^a = e^a_\mu\,\mathrm{d}x^\mu$ and $\mathrm{d}x^\mu = (c\,\mathrm{d}t, \mathrm{d}\rho, \mathrm{d}z, \mathrm{d}\varphi)$. Note that we choose the t-axis parallel to the 0-axis, the ρ-axis parallel to the 1-axis, the z-axis parallel to the 2-axis and finally the φ-axis parallel to the 3-axis. With these assumptions we also observe that these fields are static since their components do not depend on time, so the tetrad field in the matrix form becomes

$$e^a_\mu(x) = \begin{pmatrix} c & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \alpha\rho \end{pmatrix}. \tag{12}$$

The spin of the particles must be defined at each point of curved spacetime and represents the symmetry of the local reference frame. Thus, with the variation of the local reference frame, we have that the spin connection coefficients are given by the expression $\omega^a_{\mu b}(x) = e^a_\nu(x)\nabla_\mu e^\nu_b(x)$, and its non-null components in the cosmic string spacetime are

$$\omega^3_{\varphi 1}(x) = -\omega^1_{\varphi 3}(x) = \alpha. \tag{13}$$

From now on, we will analyze the dynamics of the particle. We again consider that the particle is in a circular movement of radius ρ. In this case, the metric of the cosmic string is restricted to

$$\mathrm{d}s^2 = -c^2\,\mathrm{d}t^2 + \alpha^2\rho^2\,\mathrm{d}\varphi^2. \tag{14}$$

So we have $\mathrm{d}x^\mu = (c\,\mathrm{d}t, 0, 0, \mathrm{d}\varphi)$. We can write the proper time of each particle as

$$\mathrm{d}\tau = \frac{\alpha\rho}{v}\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}} \mathrm{d}\varphi, \tag{15}$$

with $v \equiv \alpha\rho\frac{\mathrm{d}\varphi}{\mathrm{d}t}$ being the circular velocity of the particle. So the components of the 4-velocity $U^\mu(x)$ will be

$$U^t(x) = \frac{c}{\sqrt{1-\frac{v^2}{c^2}}} = c\cosh\xi, \qquad U^\varphi(x) = \frac{v}{\alpha\rho}\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{c}{\alpha\rho}\sinh\xi, \tag{16}$$

where we write the ratio v/c in terms of the rapidity parameter as $\frac{v}{c} = \tanh\xi$. Hence, the change of the local inertial referential (4) becomes

$$\delta e^3_\mu(x) = -\frac{c}{\rho}\sinh\xi\, e^1_\mu(x)\,\mathrm{d}\tau, \qquad \delta e^1_\mu(x) = \frac{c}{\rho}\sinh\xi\, e^3_\mu(x)\,\mathrm{d}\tau, \tag{17}$$

which consists in a rotation around the 2-axis in the local reference frame.

This kind of movement produces a 4-acceleration that can be attributed to any external force which keeps the particles in the circular trajectory. In the configuration above, there is only one non-null component of the 4-acceleration $a^\mu(x) = U^\nu(x)\nabla_\nu U^\mu(x)$:

$$a^\rho(x) = \frac{-c^2}{\rho}\sinh^2\xi. \tag{18}$$

So, the change in the particles' 4-momentum leads to a new direction in the movement of the particles. The momentum in the local inertial referential will obey the rules of the local Lorentz transformation [23]:

$$\Lambda^a_b(x) = \delta^a_b + \lambda^a_b(x)\,\mathrm{d}\tau, \tag{19}$$

where

$$\lambda^a_b(x) = \frac{-1}{mc^2}\{a^a(x)p_b(x) - p^a(x)a_b(x)\} - U^\nu(x)\omega^a_{\nu b}(x). \tag{20}$$

Following this, we can find the non-null components of $\lambda^0_1(x)$. This is given by

$$\lambda^0_1(x) = \lambda^1_0(x) = \frac{-c}{\rho}\cosh\xi\,\sinh^2\xi, \tag{21}$$

which represents a boost along the 1-axis. The other components represent a rotation along the 2-axis:

$$\lambda^3_1(x) = -\lambda^1_3(x) = \frac{c}{\rho}\sinh\xi\,\cosh^2\xi. \tag{22}$$

The spin precession is given in accordance with the Wigner rotation. For the infinitesimal local Lorentz transformations, the Wigner rotation is given by

$$W^a_b(x) = \delta^a_b + \vartheta^a_b(x)\,\mathrm{d}\tau, \tag{23}$$

where $\vartheta^0_0(x) = \vartheta^0_i(x) = \vartheta^i_0(x) = 0$ and

$$\vartheta^i_k(x) = \lambda^i_k(x) + \frac{\lambda^i_0(x)p_k(x) - \lambda_{k0}(x)p^i(x)}{p^0(x) + mc}. \tag{24}$$

In the presence of the cosmic string, the non-null components of $\vartheta^a_b(x)$ are

$$\vartheta^1_3(x) = \frac{c}{\rho}\sinh\xi\,\cosh\xi. \tag{25}$$

At this moment we localize the observers in the rest reference frame of the particles at the position $\varphi = \pm\Phi$ and taking a time $\tau = \alpha\rho\Phi/c\sinh\xi$, each particle reaches the respective observer and the Wigner rotation, derived from (23) by a continuous succession of infinitesimal transformations, becomes a rotation around the 2-axis [23]:

$$W^a_b(\pm\Phi,0)=\begin{pmatrix}1&0&0&0\\0&\cos\theta&0&\pm\sin\theta\\0&0&1&0\\0&\mp\sin\theta&0&\cos\theta\end{pmatrix}. \tag{26}$$

By iterating the infinitesimal transformations for a finite proper time, the Wigner rotation becomes

$$W^a_b(x_f,x_i)=T\exp\left\{\int_{\tau_i}^{\tau_f}\vartheta^a_b(x(\tau))\,\mathrm{d}\tau\right\}, \tag{27}$$

where we suppose that the particle moves along a path $x^\mu(\tau)$ from $x^\mu_i=x^\mu(\tau_i)$ to $x^\mu_f=x^\mu(\tau_f)$ and T is the time-ordering operator. This expression for the Wigner rotation gives the change of the spin when the particle is displaced in a curved background, where we can write this rotation as

$$W^a_b(\pm\Phi,0)=\exp\int\vartheta^a_b(\pm\Phi)\,\mathrm{d}\tau=\exp\vartheta^a_b(\pm\Phi)\tau. \tag{28}$$

The rotation matrix $\vartheta^a_b(\pm\Phi)$ is

$$\vartheta^a_b(\pm\Phi)=\begin{pmatrix}0&0&0&0\\0&0&0&k\\0&0&0&0\\0&-k&0&0\end{pmatrix}, \tag{29}$$

where $k=\frac{c}{\rho}\sinh\xi\cosh\xi$. Let us use the definition of the exponential form of a matrix in equation (28):

$$W^a_b(\pm\Phi,0)=1+\vartheta^a_b(\pm\Phi)\tau+\tfrac{1}{2}\left(\vartheta^a_b(\pm\Phi)\tau\right)^2+\tfrac{1}{3!}\left(\vartheta^a_b(\pm\Phi)\tau\right)^3+\cdots. \tag{30}$$

Reorganizing the odd and even power terms, we rewrite the Wigner function as

$$W^a_b(\pm\Phi,0)=\delta^a_b+\{\cos(k\tau)-1\}\vartheta^a_b(\pm\Phi)^2+\sin(k\tau)\vartheta^a_b(\pm\Phi). \tag{31}$$

Therefore, the angle of the Wigner rotation in expression (31) with $\tau=\frac{\alpha\rho}{c}\frac{\Phi}{\sinh\xi}$ will be

$$\theta=\alpha\Phi\cosh\xi. \tag{32}$$

In order to get only the influence of the relativistic effects on the spin precession spin, we must remove the spurious effects coming from trivial rotations. These spurious effects provoked by trivial rotations are eliminated through a rotation in the basis $\varphi=\pm\Phi$ around the 2-axis in the inertial local referential by an angle $\mp\Phi$, as done in [23]. Hence, the final state will not have the influence of the spurious effect:

$$\begin{aligned}\hat{U}(\Lambda(\pm\Phi))|\psi\rangle=\frac{1}{\sqrt{2}}&\left\{\cos\Delta\left(\left|p^a_+,\uparrow;\Phi\right\rangle'\left|p^a_-,\downarrow;-\Phi\right\rangle'-\left|p^a_+,\downarrow;\Phi\right\rangle'\left|p^a_-,\uparrow;-\Phi\right\rangle'\right)\right.\\&\left.+\sin\Delta\left(\left|p^a_+,\uparrow;\Phi\right\rangle'\left|p^a_-,\uparrow;-\Phi\right\rangle'+\left|p^a_+,\downarrow;\Phi\right\rangle'\left|p^a_-,\downarrow;-\Phi\right\rangle'\right)\right\}.\end{aligned} \tag{33}$$

Therefore, the angle of the Wigner rotation, when the spurious effect of the angle Φ is removed, will be

$$\begin{aligned}\Delta&=\theta-\Phi\\&=\Phi\{\alpha\cosh\xi-1\}.\end{aligned} \tag{34}$$

We note that the cosmic string topology and the particle accelerations influence the break of the anti-correlations of the spins. The spin anti-correlations are given by the position of the observers, as well as by the acceleration and by the topology of the spacetime. We point out that there is an apparent breaking in the non-local correlations in the case of measurements of the spin in the rest frame of the observer, on the 3-axis, for instance. It is an apparent breaking. Therefore, the observers can turn their apparatus of measure to an angle $\mp\theta$ around the 2-axis of their local inertial referential and again recover the perfect anti-correlation of spins.

Now we consider two special limits. The first case is the accelerated movement in the Minkowski spacetime $\alpha = 1$. In this case, we obtain the following expression:

$$\left\{\vartheta_1^3(x) - \chi_1^3(x)\right\} \mathrm{d}\tau = -\left\{\frac{c}{\rho}(\cosh\xi - 1)\sinh\xi\right\} \mathrm{d}\tau.$$

In the non-relativistic limit $v/c \ll 1$, we have

$$\left\{\vartheta_1^3(x) - \chi_1^3(x)\right\} \mathrm{d}\tau \approx -\frac{1}{2}\frac{v^2}{c^2}\,\mathrm{d}\phi. \tag{35}$$

The difference between the angles $\vartheta_1^3(x) - \chi_1^3(x)$ gives rise to the Thomas precession as pointed out by Terashima and Ueda [23]. The second case is the limit where $\alpha \neq 1$ and non-relativistic case $v/c \ll 1$, yielding

$$\Delta = \Phi\alpha\left\{\frac{1}{2}\frac{v^2}{c^2} + \left(\frac{\alpha - 1}{\alpha}\right)\right\}. \tag{36}$$

In this limit, we obtain the contribution of the acceleration due to the topological defect in a sum of two terms. In the case where $\alpha = 1$, we observe that the effect of the rotation is provided solely by the acceleration term. Note that the term due to the topology of space contributes to increase the angle Δ if $\alpha > 1$ and decrease it if $\alpha < 1$. The cosmic string geometry parameter $0 < \alpha < 1$. The case where $\alpha > 1$ is the anti-cone case.

4. Conclusion

In this paper, we investigate the EPR correlations in the spacetime with a topological defect. We note that the presence of the cosmic string apparently breaks the spin anti-correlations of the EPR pairs. Observing the result (34), we see that the presence of the cosmic string deteriorates the perfect anti-correlation in the same direction of the initial configuration, in the local reference frame of the observers, as was pointed by Terashima and Ueda [23]. Note that in the case where the parameter $\alpha = 1$ in equation (34), we have the Minkowski limit. In this case, the apparent deterioration of the EPR correlation is due to the acceleration of the particles. Analyzing the presence of the cosmic string, the parameter α contributes to decrease the angle of the spin precession in relation to the Minkowski case because this parameter assumes only values in the range $0 < \alpha < 1$. In the anti-cone case, the spin precession angle increases due to the fact that $\alpha > 1$. The perfect correlation can be obtained if we take into account the relativistic effect arising from acceleration and topology of the spacetime in order to correct the direction of the spin axis of measurements. We can explore perfect anti-correlation for quantum communication by rotating the direction of measurement in such a way as to compensate the direction of measurement at $\pm\Phi$ by a rotation about the 2-axis through the angles $\mp\theta$. It is important to stress that our results differ from those obtained by Terashima and Ueda. While their results come from the curvature of the spacetime, ours are of a topological origin, since locally the cosmic string curvature vanishes everywhere, except in the origin.

Acknowledgments

We are indebted to Professor F Moraes for the critical reading of the manuscript. This work was partially supported by CNPq, CAPES (PROCAD), PRONEX/CNPq/FAPESQ and CNPq/Universal.

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